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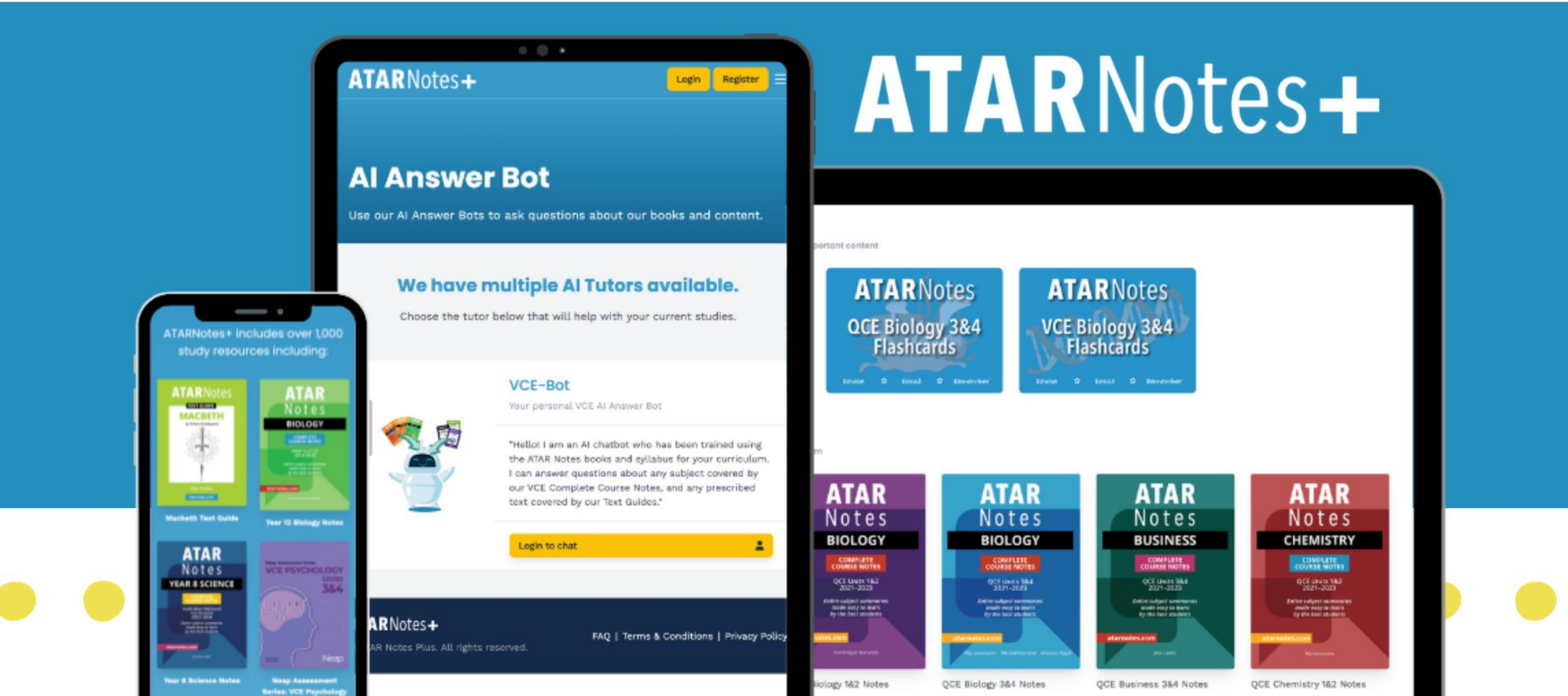


ATARNotes

Specialist Maths 34

ATARNotes January Lecture Series

Presented by:
Manjot Bhullar

About me!

- Hey, everyone my name is Manjot Bhullar
- Bachelor of Biomedical Science
- ATAR of 99.80
- Maths Tutor at Tutesmart
- The subjects I did throughout VCE
  - Chemistry
  - Maths Methods
  - Specialist Maths
  - English
  - Biology
  - Further Maths

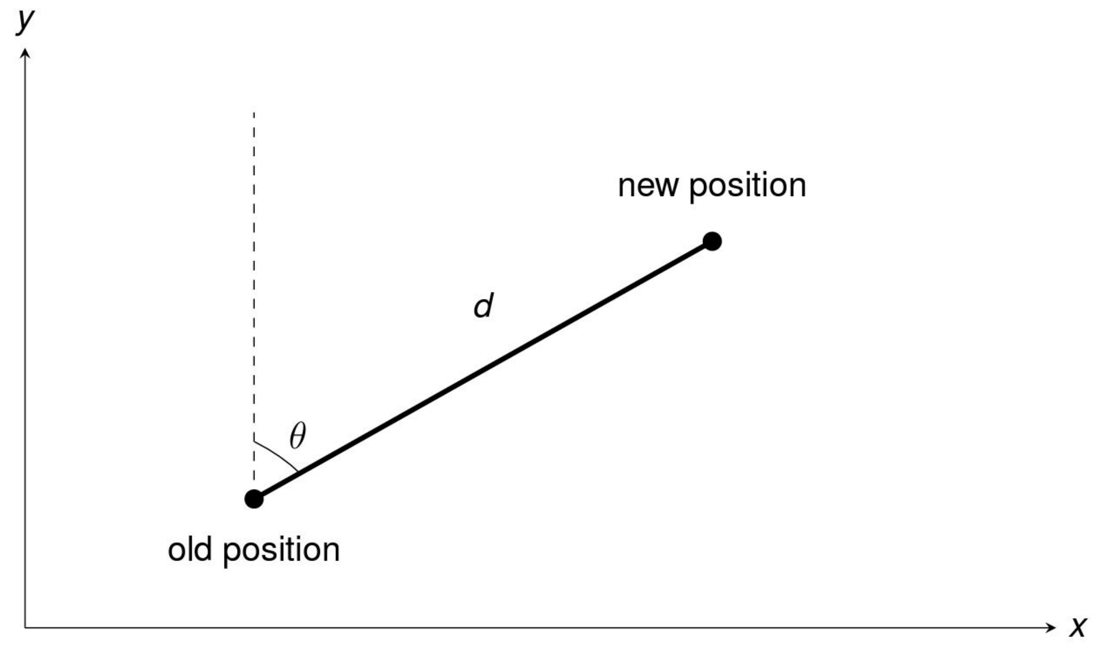
- Scales REAL REAL good, so great investment of time and energy
- This is a subject that requires academic resilience, patience and commitment
- With enough practice and exams, it's a very <u>predictable</u> subject
  (Examiners know it's a hard subject)
- Don't be intimidated! (high school is more about sheer effort rather than 'being good with numbers' or 'high IQ'.)
- You will either absolutely hate or absolutely love this subject but do try going into this subject with a positive mindset.

Overview

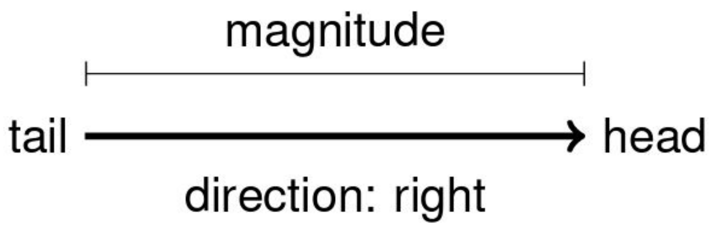
- Vectors and Vector Calculus
- Complex Numbers
- Logic and Proof
- Functions and Relations
- Kinematics
- Differential and Integral Calculus
- Differential Equations
- Probability
- Hypothesis Testing and Statistics

What are Vectors?

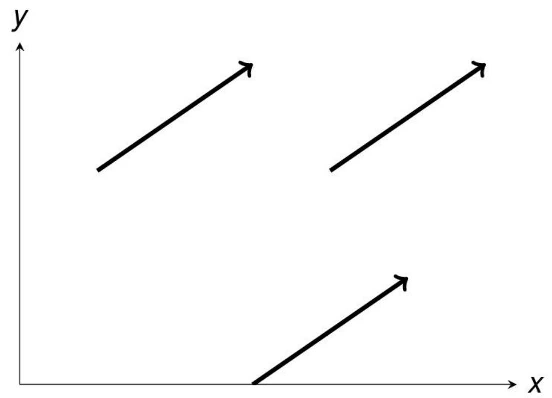
- Vectors are mathematical quantities consisting of both a magnitude and direction
- We use vectors to describe change in position



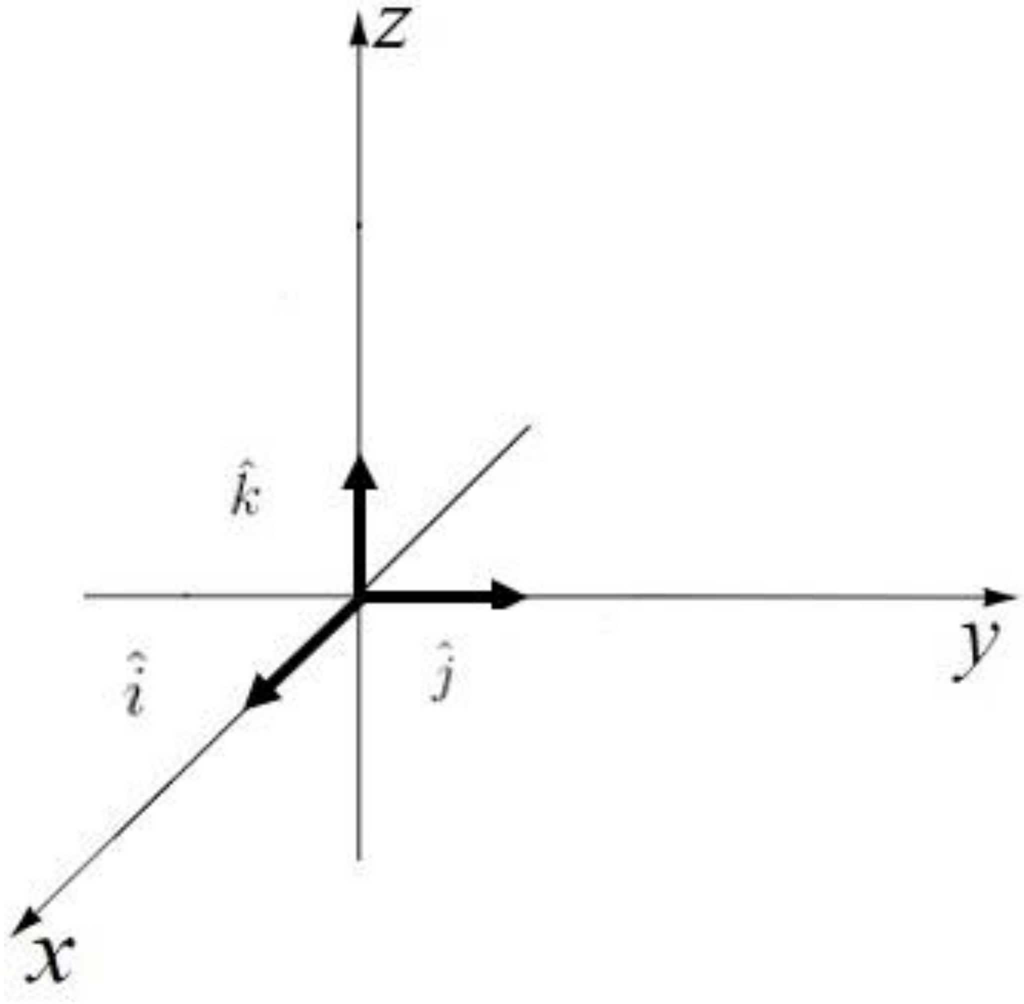
Depicting Vectors



- Vectors exist in 3 dimensions and two vectors are the same if they have the same magnitude and direction despite having different starting points in space

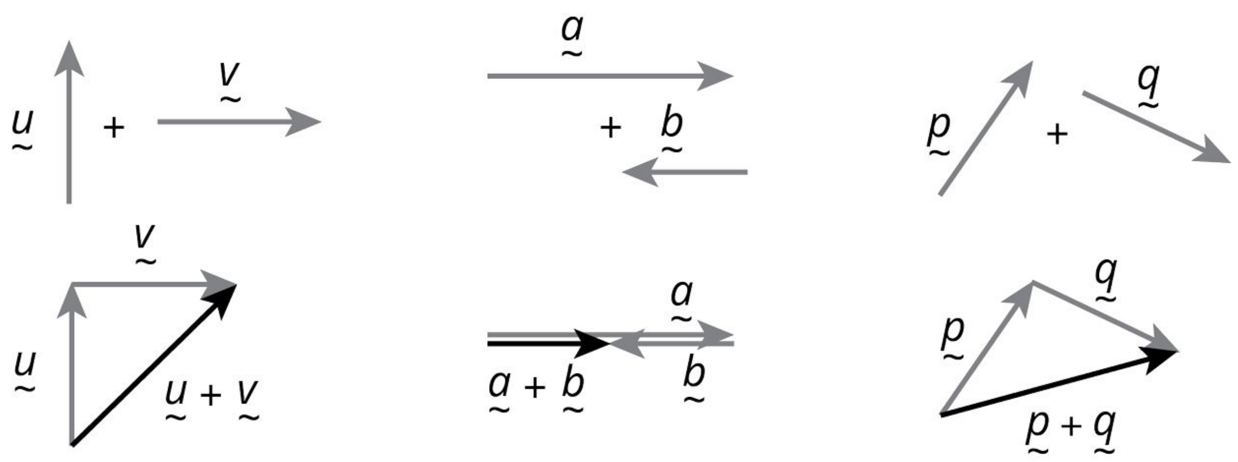


Expressing Vectors



Adding Vectors

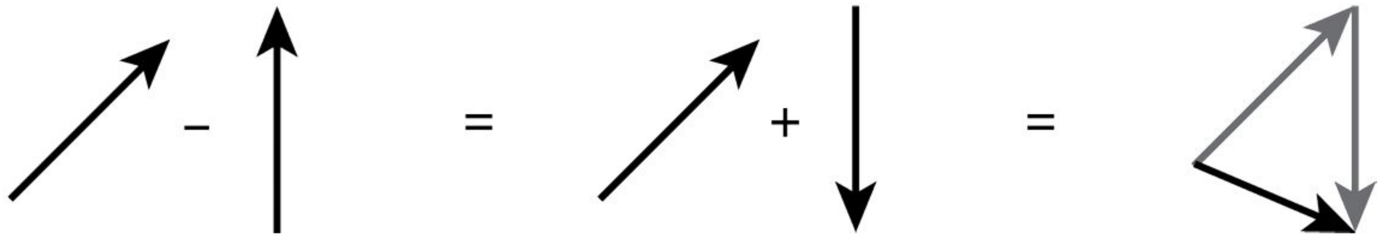
1. We take the two vectors that we want to add.
2. We align them so that the head of the first vector touches the tail of the second.
3. We draw an arrow from the tail of the first vector to the head of the second.



$$\left(3\underset{\sim}{i}+7\underset{\sim}{j}-2\underset{\sim}{k}\right)+\left(\underset{\sim}{i}+2.5\underset{\sim}{j}+11\underset{\sim}{k}\right)=4\underset{\sim}{i}+9.5\underset{\sim}{j}+9\underset{\sim}{k}$$

Vector Subtraction

- Follow the exact same process as Vector Addition but reverse the direction of the vector that is being subtracted



Length/Magnitude of Vector:

- If a vector is r = xi + yj + zk:

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

Parallel Vectors:

- Two vectors, $\vec{u}$ and $\vec{v}$, are **parallel** if $\vec{u} = k\vec{v}$ where k is a scalar (this stretches/squishes the magnitude).

Unit Vectors:

- Special Vectors that have a magnitude of 1 to SPECIFY direction. We just divide the vector by its magnitude.

$\vec{u}$

Scalar (Dot) Product

When we multiply vectors together = we get a scalar!

Super simple: Remember to multiply like and like together and add. Eg:

$$\vec{a} = a_1\vec{i} + b_1\vec{j} \qquad \vec{b} = a_2\vec{i} + b_2\vec{j}$$

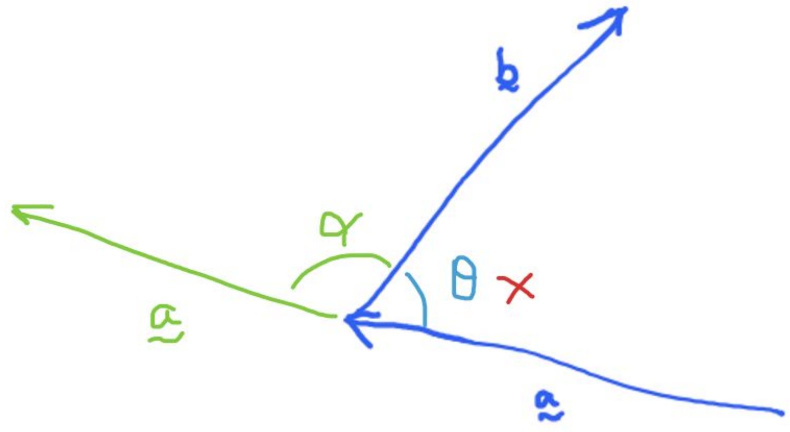
Their dot product is:

$$\vec{a}\cdot\vec{b} = a_1a_2 + b_1b_2$$

Vector Angles

To find angles between 2 vectors, THEY <u>MUST</u> BE TAIL TO TAIL

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$



SOME IMPORTANT PROPERTIES

- $\vec{a}\cdot\vec{a} = |a|^2$
- $\vec{a}\cdot\vec{b} = 0$ if $\vec{a}$ and $\vec{b}$ are perpendicular

Cross Product

Given two vectors $\underset{\sim}{a} = a_1 \underset{\sim}{i} + a_2 \underset{\sim}{j} + a_3 \underset{\sim}{k}$ and $\underset{\sim}{b} = b_1 \underset{\sim}{i} + b_2 \underset{\sim}{j} + b_3 \underset{\sim}{k}$, the cross product is defined as:

$$\underset{\sim}{a} \times \underset{\sim}{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$$

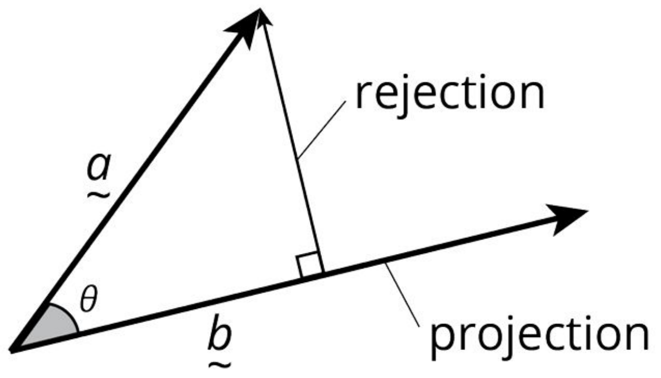
Example

- Find the cross product of u= 3i + 2j - k and v= 4i - 6j + k

$$\underset{\sim}{u} \times \underset{\sim}{v} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 4 \\ -6 \\ 1 \end{pmatrix} = \begin{pmatrix} (2)(1) - (-1)(-6) \\ (-1)(4) - (3)(1) \\ (3)(-6) - (2)(4) \end{pmatrix} = \begin{pmatrix} -4 \\ -7 \\ -26 \end{pmatrix}$$

$$\underset{\sim}{u} \times \underset{\sim}{v} = -4\underset{\sim}{i} - 7\underset{\sim}{j} - 26\underset{\sim}{k}$$

Projections

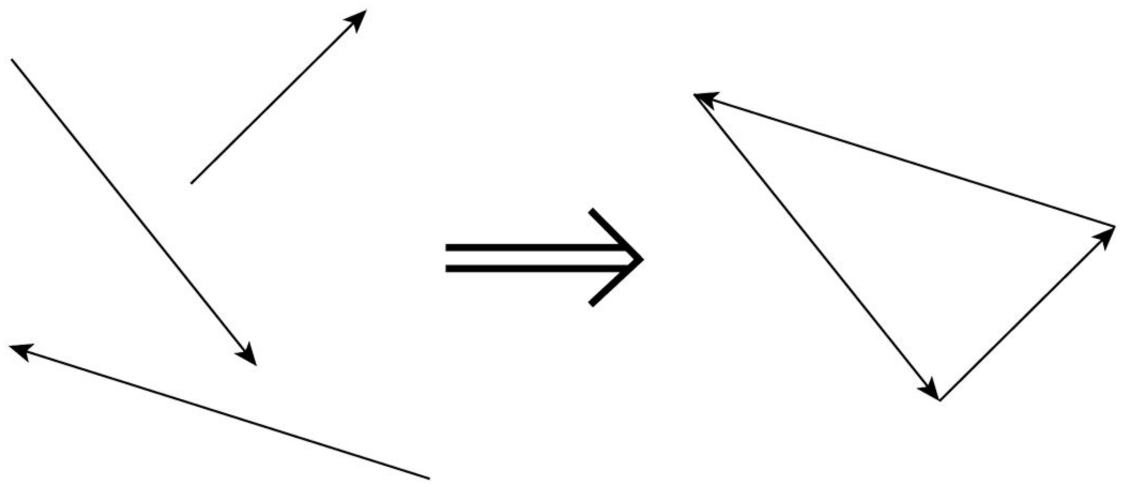


- The scalar resolute (magnitude of the projection) can be determined by: $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$
- The vector resolute (projection vector) of $\vec{a}$ in the direction $\vec{b}$ is: $\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}| \cdot |\vec{b}|}\vec{b}$
- The Rejection (perpendicular) vector can be determined by: $\vec{a} - \vec{u}$

- Find the vector resolute of $\vec{a}$ in the direction of $\vec{b}$ given:

$$\vec{a} = \vec{\imath} - 3\vec{k} \quad \vec{b} = \vec{\imath} - 4\vec{\jmath} + \vec{k}$$

- A set of vectors are linearly dependent if the vectors align on a single linear plane
  - That is $k_1 a + k_2 b + k_3 c \neq 0$



Determine whether the following vectors are linearly independent

a = 3i - 2j + k, b = 6i - 3j + 5k and c= 4i + 5j - 2k

We are trying to see if k_1 a + k_2 b + k_3 c = 0

$$\begin{bmatrix} 3 & -2 & 1 \\ 6 & -3 & 5 \\ 4 & 5 & -2 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Linear dependency will occur when there is no unique solutions

$\det\left(\begin{bmatrix} 3 & -2 & 1 \\ 6 & -3 & 5 \\ 4 & 5 & -2 \end{bmatrix}\right)$ = -79. As determinant is not 0, the set of vectors are linearly

Complex Numbers

Professor: $\sqrt{-1}$ is not real

Complex Numbers

Complex Numbers consist of a real (x) and imaginary (y) component

- Can be expressed in the form z = a + bi

$$i = \sqrt{-1}$$

$$i^2 = -1$$

$$i^3 = i^2 \times i = -i$$

$$i^4 = i^2 \times i^2 = 1$$

$$i^5 = i^4 \times i = i$$

Complex Conjugate

- The **complex conjugate** is denoted by $\bar{z}$, and is given by:

$$z = x + yi \qquad \rightarrow \qquad \bar{z} = x - yi$$

Some properties:

$$z\bar{z} = x^2 + y^2 = |z|^2$$

Simplify the following:

a. $2i(i+3)$

b. $(2+i)(32-i)$

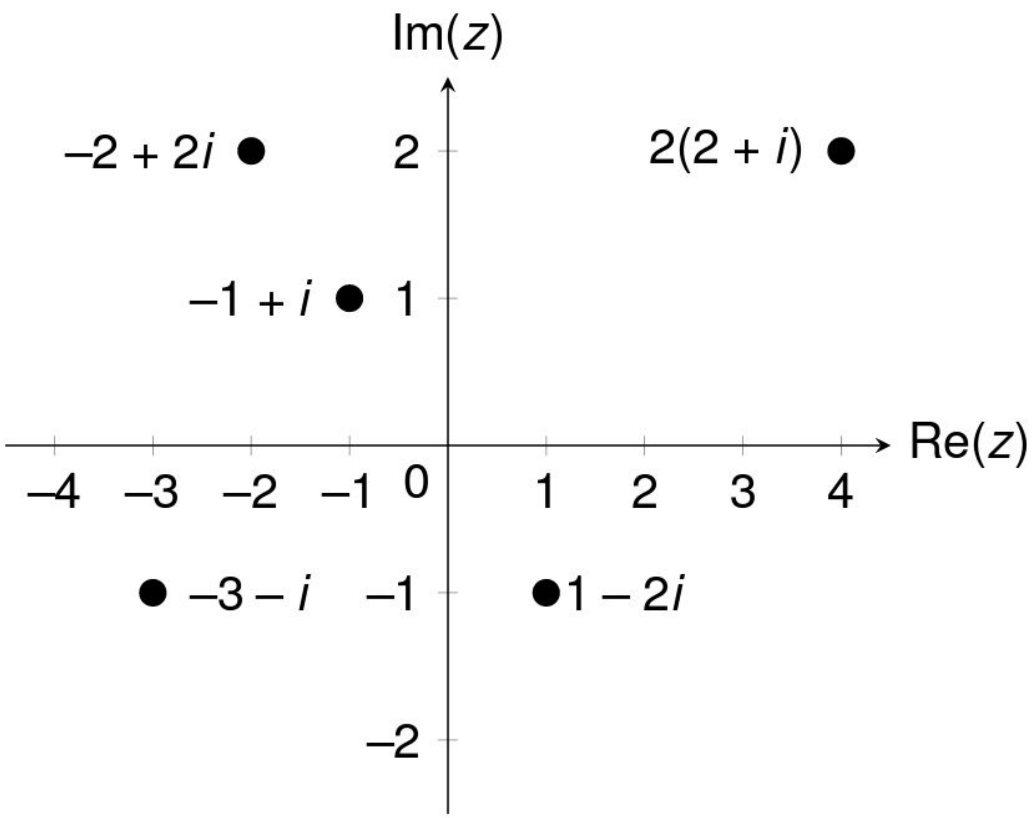
c. $\frac{1+i}{2+i}$

Solve the following:

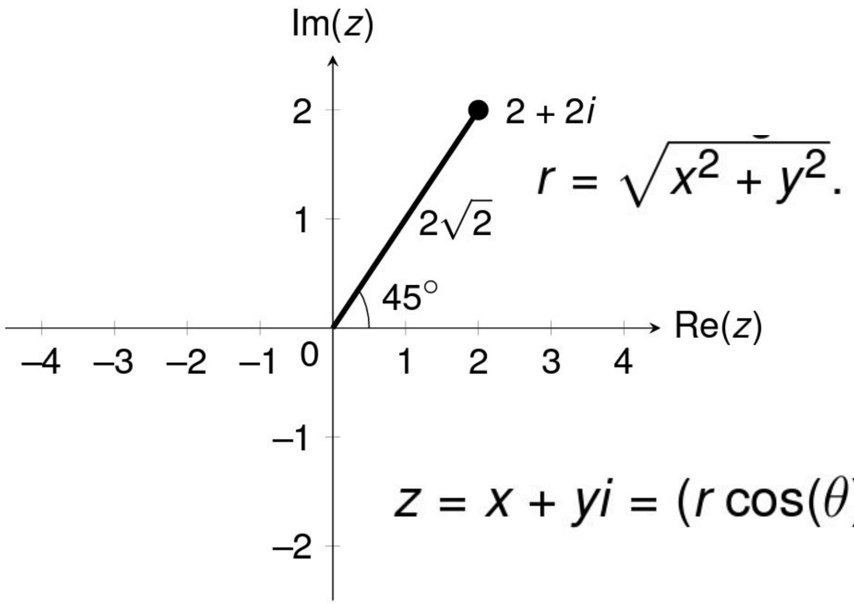
a. $x^2 + 16 = 0$

b. $x^2 + 4x + 5 = 0$

Argand Diagram



- Polar form is a way of expressing a complex number in terms of an angle and length as opposed to the typical (x,y) coordinates
- This angle is measured counter-clockwise from the positive x-axis, and the length is the distance from the origin



SUPER important: DOMAINS

If you are given Arg (not arg) the domain is $\text{Arg}(\theta) \in (-\pi, \pi]$

You have to draw the CAST quadrants

$$z = x + yi = (r\cos(\theta)) + (r\sin(\theta))i = r(\cos(\theta) + i\sin(\theta)) = r\text{cis}(x)$$

Convert to Polar Form:

a. $1 + i$

b. $23i$

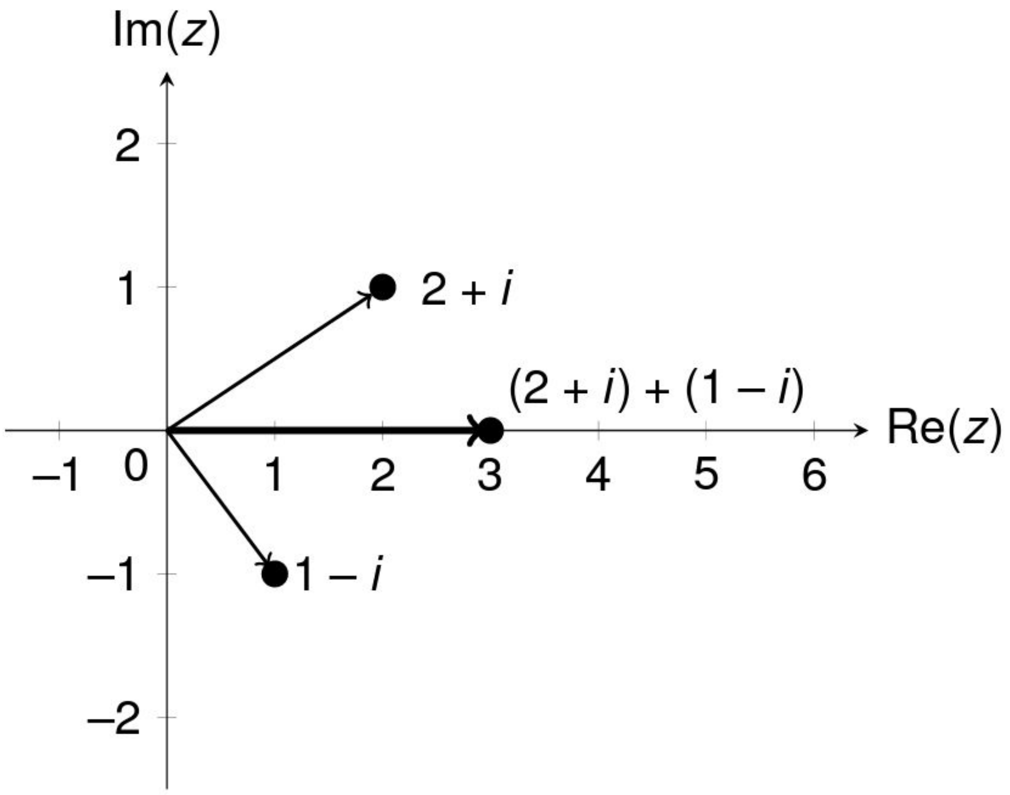
c. $\sqrt{3} + 3i$

Convert to Cartesian Form:

a. $4cis(2\pi)$

b. $8cis\left(\frac{25\pi}{3}\right)$

- To add or subtract complex numbers, just add or subtract the real and imaginary separately (collecting like terms)



- Rule: multiply/divide to r add/subtract θ

Consider two complex numbers: (USE POLAR FORM FOR THIS)

$$z_1 = r_1 cis(\theta_1) \quad z_2 = r_2 cis(\theta_2)$$

Multiplication:

$$\boxed{Arg(z_1 z_2) = Arg(z_1) + Arg(z_2)}$$

$$z_1 \cdot z_2 = r_1 r_2 cis(\theta_1 + \theta_2)$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2)$$

Conjugate:

$$\bar{z} = rcis(-\theta)$$

Reciprocal:

$$z^{-1} = \frac{1}{r} cis(-\theta)$$

Simplify. Give answer in Polar Form

a. $4cis\left(\frac{5\pi}{2}\right) \cdot 13cis\left(\frac{\pi}{6}\right)$

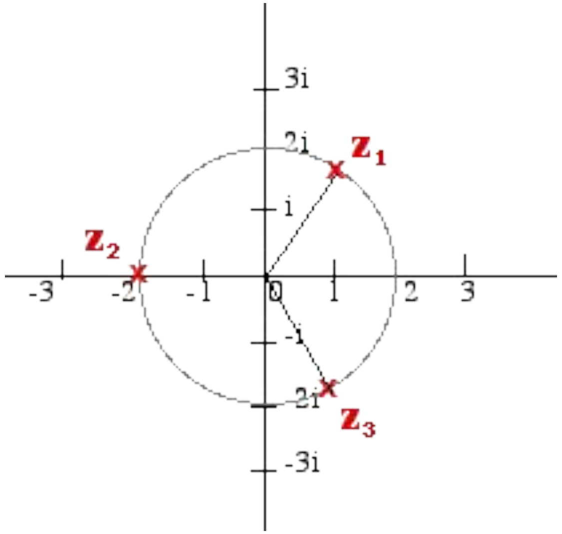
- We use this for finding nth roots of a complex number

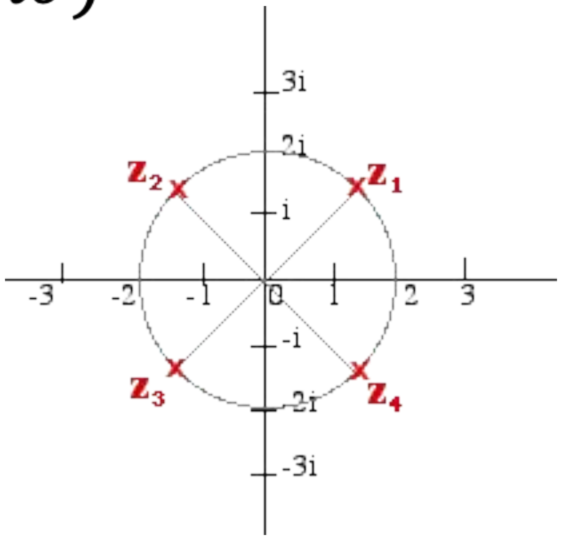
(eg. Numbers have can 2 square roots, 3 cube roots, etc……)

Therefore: There are n solutions for nth roots around a circle

MUST BE IN POLAR FORM AND IN $Arg(\theta)\epsilon(-\pi,\pi]$

$$z^n = r^n cis(n\theta)$$





- 1. Write the formula: $z^n = r^n cis(n\theta)$
  2. Convert the complex number into Polar form
  3. Let r^n = magnitude and $n\theta$ = arg

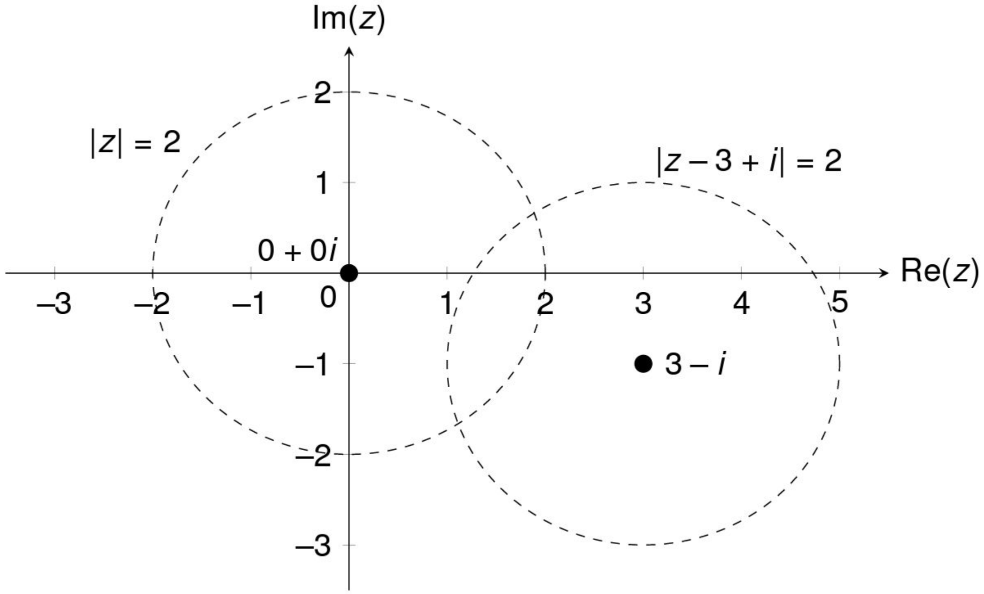
     For angles if you find one angle, just $\pm$ angles equally apart
  4. Make sure $Arg(\theta)\epsilon(-\pi,\pi]$
  5. Convert into cartesian if needed

$$z = r^{\frac{1}{n}} \text{cis}\left(\frac{\theta + 2k\pi}{n}\right) \text{ where } k \text{ is } 0, 1, \ldots, n-1$$

Solve: $z^4 = 2 + 2i$

- Relations on the complex plane are a set of points that satisfy a certain condition (locus)
  - For example, the condition $|z| = 2$, includes all the numbers on the complex plane that have a magnitude of 2 => a circle centred at 2

$$|z - 3 + i| = 2$$



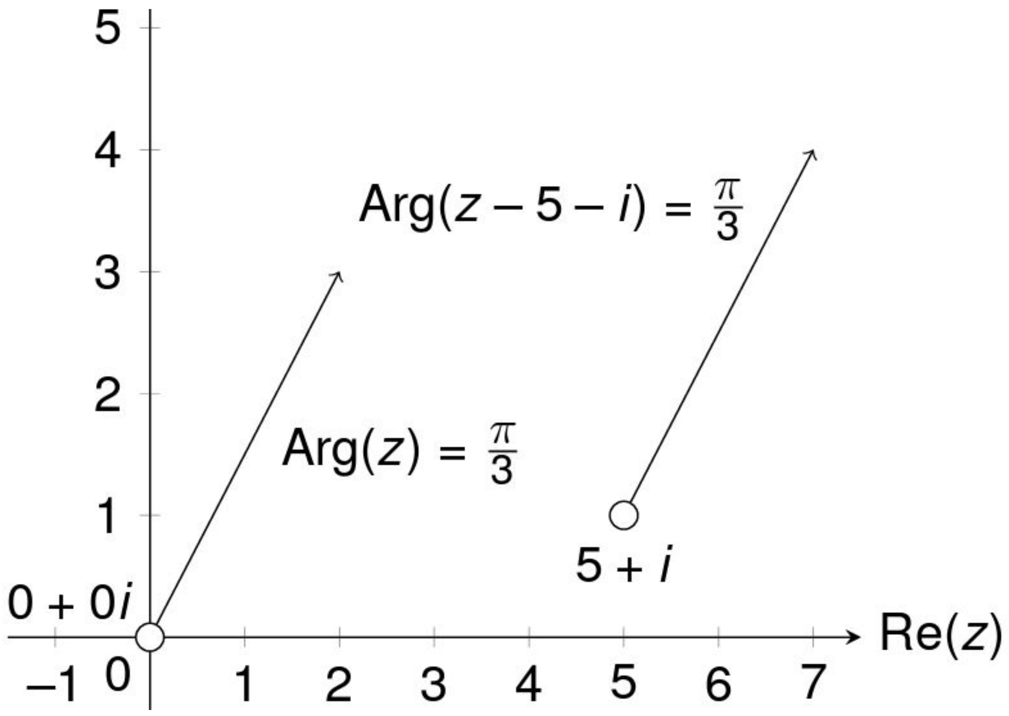
Circles

$$|z - a| = r$$

Rays

$$Arg(z - b) = \theta$$

(ALWAYS an OPEN DOT at origin)



<u>Lines</u> (NEVER do this algebraically unless told to)

$$|z - a| = |z - b|$$

Also known as perpendicular bisectors:

1. Graph 2 points given
2. Join them together with dotted line
3. Use midpoint and gradient formula between the points
4. Get the normal and apply to $y - y_1 = m(x - x_1)$
5. Draw perpendicular line from midpoint

Draw:

a. $|z + 1 - 3i| = 4$

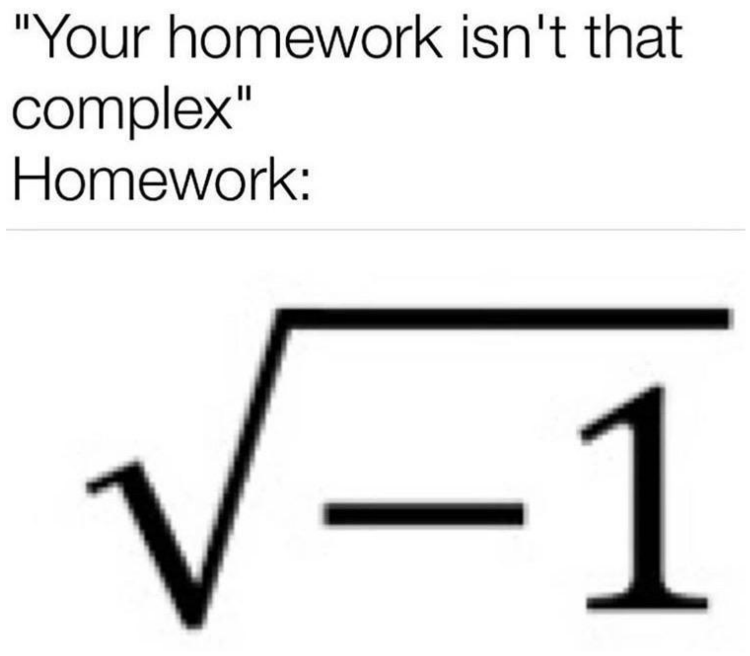
b. $Re(Z) - Im(Z) = 5$

c. $Arg(Z) > \frac{\pi}{3}$

Overview

Today we covered:

Vectors:
Additions, Dot products, Linear Dependence, Vector Resolutes



Complex Numbers:
Argand diagrams, Polar forms, De Moivres, Regions

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QUESTIONS?